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INCOME MOBILITY, PERMUTATIONS, AND RERANKINGS.

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Abstract

Chakravarty, Dutta and Weymark (1985) present axioms for an ethical index of relative income mobility in a two period world. This paper presents a decomposition of this index into two terms: (i) an index of structural mobility which captures differences in the inequality of the cross-section income distributions, and (ii) an index of exchange mobility which captures changes in relative incomes. These concepts are shown to be useful in the evaluation of an income tax system which induces rerankings between the pre-tax and the after-tax income distributions, as well as in other contexts where there are reorderings between individuals.

Keywords: Income mobility ; income inequality ; rerankings; horizontal inequality; equivalence scales.

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Abstract

Chakravarty, Dutta and Weymark (1985) present axioms for an ethical index of relative income mobility in a two period world. This paper suggests a decomposition of this index into two terms: i) an index of structural mobility which captures differences in the inequality of the cross-section income distributions, and ii) an index of exchange mobility which captures changes in relative incomes. These concepts are shown to be useful in the evaluation of an income tax system which induces rerankings between the pre-tax and the after-tax income distributions, as well as in other contexts where there are reorderings between individuals.

INTRODUCTION

Compared to agrarian society, which has occupied most of historical times, our growth-oriented industrial society is presumed to be socially mobile and egalitarian(1). The recent availability of longitudinal data makes increasingly possible the measurement of such a central concept as mobility. The problem is that, compared with the neighboring area of inequality measurement, there is less professional agreement about how to measure this dynamic concept.

Social mobility is, of course, a many-sided phenomenon. Among the approaches developed by economists, we find it useful to distinguish between two types. The first approach considers explicitly the transition mechanism responsible for the time path of the variable of interest. Such mechanism is often represented by a transition matrix which shows the fraction of the population which moves from one category to another in one time period. In this context, an index of mobility is defined as a real function on the set of transition matrices(2). Alternatively, mobility measures are also derived from other simple stochastic specifications of the transition mechanism(3).

As pointed out in Shorrocks (1978b), these attempts are mainly concerned with stock variables, interpreted to include social status and occupation as well as wealth and the assets of firms. The second approach, which we follow in this paper, is meant for a less abstract setting where the variable of interest is income. Abstracting from the transition mechanism, one simply studies in a straightforward way the changes that can be observed in longitudinal data sets: changes in cross-section income inequality and changes in relative incomes or in absolute income differences. Indices of relative or absolute mobility are sensitive to changes in relative incomes or in income differences, respectively.

We can distinguish between descriptive and normative income mobility measures(4). Naturally, descriptive measures cannot tell us whether mobility is or is not socially desirable. In this paper, we are concerned with ethical indices of relative income mobility which are capable of addressing this issue.

Ethical indices are derived from explicit social evaluation functions (SEF, for short). In a static context, the SEF is simply defined on the space of one-period income distributions. In the present dynamic context, what the SEF domain should be is not an obvious question. Given a decision in this regard, it is important to know whether in order to construct meaningful mobility measures we need SEFs which incorporate new value judgments.

In his seminal contribution in this area, King (1983) proposes a two period model where the SEF is defined on individual incomes during the second period *and* rank reversals between the two periods. Therefore, new value judgments about the welfare effects of rank reversals are required. In this paper, we follow the ethical approach due to Chakravarty, Dutta and Weymark (1985), or CDW for short. They compare the actual time path of incomes received over a number of periods with a hypothetical benchmark which maintains constant over time the relative positions occupied by the individuals in the actual first-period income distribution. For operational reasons, CDW also restrict themselves to a two period model. Contrary to King (1983)'s, CDW's SEF is defined on aggregate incomes over the two periods and does not include any new value judgment beyond the traditional ones.

In this framework, we find it essential to distinguish between two types of rank reversals ignored in CDW: rank reversals between the first- and second-period income distributions, which we call *permutations;* and rank reversals between the first-period and the aggregate income distributions, which we call *rerankings.* The distinction can be illustrated by means of a pair of simple examples. In both examples the first period income distribution is (2, 4). In example 1, the second period income distribution is (4, 3). Therefore, there is a permutation, but since the aggregate income distribution is (6, 7), there is no reranking. In example 2, the second period income distribution is (7, 0), representing the same total income growth as before. The aggregate income distribution is now (9, 4), so there is both a permutation and a reranking.

Using this distinction, we offer a novel decomposition of a version of CDW's relative mobility index into two terms: the first term, which we call *structural mobility,* captures the welfare effect of the change in inequality between the aggregate and the completely immobile distribution, once all permutations have been eliminated. The second term, which we call *exchange mobility,* measures the welfare impact of permutations between the first- and the second-period income distributions, with or without rerankings between the initial and the aggregate income distributions. We do not impose any value judgments either on permutations or rerankings. However, in the presence of permutations, we show that exchange mobility is always socially desirable. On the other hand, in the presence of rerankings, we show that there exists a second-period income distribution which implies the same rate of income growth, the same income mobility, but no rerankings at all.

These ideas, developed in an income growth context, have an immediate application to the evaluation of income tax systems and other interesting problems.

The paper is organized in five sections. The first two sections are devoted to a discussion of the assumptions and the decomposition of the income mobility model in the homogeneous case. The third section applies our decomposition of overall mobility into structural and exchange mobility in an income tax context. The fourth section briefly reviews other applications, while the fifth section concludes.

I. THE MODEL IN THE HOMOGENEOUS CASE

Let the time interval $[t_0,\ldots,t_m)$ be partitioned into m equal subperiods $[t_{k-1},\ldots,t_k)$, $k = 1,\ldots,m$, where m is a fixed exogenous integer. We refer to $[t_{k-1},\ldots,t_k)$ as the kth-period. Let there be $i = 1,\ldots, n$ individuals. For period k, let y_k^i be individual's i income. The income distribution in period k is denoted by $y_k = (y_k^1,\ldots,y_k^n)$. Let $D = R_{++}^n$ be the strictly positive orthant in n-dimensional

Euclidean space. Sequences of income distributions, $Y = (y_1,...,y_m) \in D^m$, are called income structures.

Each individual i is characterized by an income stream $y^i = (y_1^i ,...,y_m^i)$ Over the whole time interval $[t_0,...,t_m)$, individual i receives aggregate income $y_a^i = S(y^i)$. One of the simplest aggregation functions is the one used in Shorrocks (1978a), $S(y^i) = \Sigma_k \alpha_k y_k^i$ with α_k denoting the common weight given to every individual income in period k, and $\Sigma_k \alpha_k = 1$(5). We refer to the income distribution $y_a = (y_a^1 ,...,y_a^n)$ as the aggregate distribution.

The ethical approach to measuring income mobility in CDW uses an intertemporal social evaluation function (SEF for short), $W: D^m \rightarrow R^1$, where $W(Y)$ is the social welfare level associated with the income structure Y. The income mobility concept we wish to explore is the one embodied in a welfare comparison of the actual income structure Y and a hypothetical benchmark structure Y_b: the income structure which would have resulted in the absence of mobility given the first period distribution y_1. That is to say, mobility indices are obtained by comparing the actual level of social welfare $W(Y)$ with the level of social welfare $W(Y_b)$ which would have obtained with the benchmark structure Y_b.

To make this comparison operational, CDW make the following two fundamental assumptions referring to the notion of complete immobility in the relative case and the nature of the SEF W (.), respectively.

A. 1. Given $\mathbf{Y} = (y_1,...,y_m)$, let $\mu(y_k)$ be the mean of the income distribution in period k. We say that Y exhibits *complete relative immobility* if individual income shares are maintained through time equal to the income shares in the first period distribution y_1, i.e.

$$Y_b = (y_{b1},...,y_{bm}) = (y_1, y_1\, \mu(y_2)/\, \mu(y_1),..., y_m \mu(y_m)/\, \mu(y_1)),$$

so that $\mu(y_{bk}) = \mu(y_k)$ for all k. Consequently, the aggregate distribution for this benchmark structure, denoted by y_b, gives each individual the same share of actual total income as they receive in period 1(6).

The only features of the income structures Y and Y_b relevant for the welfare comparisons are their aggregate distributions y_a and y_b. Formally:

A. 2. There exists a SEF W: $D \rightarrow R^1$ such that

$$W(y_a) = W(Y) \text{ for all } Y \text{ in } D^m.$$

A mobility index assigns a mobility value to each income structure Y in D^m, i.e. it is a function M: $D^m \rightarrow R^1$. CDW suggest a class Ω of indices of relative mobility of the form

$$M^*(Y) = \phi\{W(y_a)/\,W(y_b)\}, \qquad (1)$$

where $\phi: R^1_{++} \rightarrow R^1$ is a continous increasing function with $\phi(1) = 0$(7). Indices in this class are ordinally equivalent to each other and to the ratio of the actual aggregate distribution welfare level to the aggregate distribution welfare level in the hypothetical immobile benchmark structure Y_b. The normalization employed ensures that an immobile income structure is assigned a mobility value of zero.

Assumption **A. 2.** is, of course, questionable. Consider the following example taken from CDW. There are two income structures, $\mathbf{Y} = \{(2,2), (2,2), (2,2)\}$ and $\mathbf{Y}^{\#} = \{(2,2), (3,1), (1,3)\}$. The first structure exhibits no mobility, while the second one also exhibits no mobility for any mobility index derived from an intertemporal SEF satisfying **A. 2.** and the following specification of the aggregation function S(.): $y_a^i = \Sigma_k \alpha_k y_k^i$ where $\alpha_k = 1$ for all k[(8)]. This example helps to show that the acceptability of **A. 2.** may depend on the specification of S(.). In this paper we want to emphasize those aspects of income mobility which do not depend on the aggregation function S(.). Therefore, as in CDW we restrict ourselves to the two period case. In this context, **A. 2.** is a more acceptable restriction. Formally, we adopt:

A. 3. We assume that $m = 2$ and $y_a^i = y_1^i + y_2^i$ for each i.

Both periods are then reflected in the construction of the mobility indices, the first-period distribution through its effect on the aggregate benchmark distribution y_b, and the second-period distribution through its effect on the actual aggregate distribution y_a.

It is a natural strategy to pay attention in the first place to the simplest but relevant case. We hope that the applications we present in this paper justify the interest of the two period case. On the other hand, recall that this is the case assumed also by other authors: for instance, King (1983) and Markandya (1984) within the normative approach,

The next assumption refers to the welfare evaluation of one-period incomes.

A. 4. There exists a SEF defined on one-period incomes, W_k: $D \rightarrow R^1$, and this function is the same as the m-period SEF W, i.e.

$$W_k(y_k) = W(y_k), \text{ for both } k = 1, 2.$$

The identification of one-period evaluations with m-period ones is also questionable but it greatly simplifies our work. Assumption A. 4. is taken also from CDW and, again, it is probably more acceptable in the two period case(9).

Among the members of the class Ω, CDW point out that one mobility index stands out because of its simple interpretation. This index is obtained by setting $\phi(s) = s - 1$ in (1). In this case, we have that

$$M(Y) = \{W(y_a) - W(y_b)\} / W(y_b). \qquad (2)$$

The remaining properties of $M(\cdot)$ depend on additional assumptions on $W(\cdot)$. For our analytical purposes, in the relative case we only need that $W(\cdot)$ can be expressed solely in terms of two statistics of the income distribution, the mean and a scale invariant index of relative inequality $I^R(\cdot)$; that is, we only need that there exists a function V: $R^2 \rightarrow R^1$ such that

$$W(y) = V(\mu(y), I^R(y)) \tag{3}$$

with $V(\cdot)$ increasing in the first argument and decreasing in the second one. Wee say that $W(\cdot)$ is regular if it is continous and S-concave. When $W(\cdot)$ is regular, Dutta and Esteban (1992) show that equation (3) is satisfied if and only if W(.) is increasing along rays from the origin and weakly-homothetic(10).

However, for operational purposes it is convenient to specify the trade-off between efficiency and distributional considerations. Consequently, we adopt:

A. 5. The SEF $W(\cdot)$ can be expressed as: $W(y) = \mu(y)(1 - I(y))$.

Thus, social welfare is seen to be the product of the mean and an adjustment factor which varies inversely with an appropriate index of relative inequality $I(\cdot)$. For example, CDW assume that $W(\cdot)$ is homothetic. In this case, it is well known that we can write

$$W(y) = \mu(y)\{1 - I^{AKS}(y)\},$$

where $I^{AKS}(\cdot)$ is the relative inequality index obtained according to the Atkinson-Kolm-Sen procedure which uses the notion of an equally distributed income(11). Alternatively, because of its good additive separability properties we may use

$$W(y) = \mu(y)\{1 - I_1(y)\},$$

where $I_1(y)$ is the first relative inequality index suggested by Theil[12]. In either case, the CDW mobility index defined in (2) becomes

$$M(Y) = \{I(y_1) - I(y_a)\}/\{1 - I(y_a)\}, \tag{4}$$

with $I(\cdot)$ equal to $I^{AKS}(\cdot)$ or $I_1(\cdot)$[13]. In what follows, we will assume that $1 - I(y_a) > 0$.

II. STRUCTURAL AND EXCHANGE MOBILITY

Contrary to descriptive mobility indices, our ethical index allows us to determine whether the observed income changes are socially desirable. Consider the following two examples. In the first one, denoted by **E1**, $\mathbf{Y} = \{(2, 4), (4, 3)\}$ and $y_a = (6, 7)$. In the second example, denoted by **E2**, $\mathbf{Y}^{\#} = \{(2, 4), (2, 5)\}$ and $y_a{}^{\#} = (4, 9)$. The initial situation is the same in both examples, $y_1 = y_1{}^{\#} = (2, 4)$. Since $\mu(y_2) = \mu(y_2{}^{\#}) = 7/2$, the rate of income growth is also the same in **E1** and **E2**. However, it is clear that

$$M(Y) = \{I(2, 4) - I(6, 7)\}/\{1 - I(2, 4)\} > 0$$

while

$$M(Y^{\#}) = \{I(2, 4) - I(4, 9)\}/\{1 - I(2, 4)\} < 0.$$

The reduction in inequality in y_a relative to y_1 causes $M(Y)$ to be positive, reflecting an increase in social welfare. But the increase in inequality in $y_a{}^{\#}$ relative to $y_1{}^{\#}$ causes $M(Y^{\#})$ to be negative, reflecting a social welfare loss.

Apparently, our mobility index reflects welfare changes due solely to changes in inequality from the initial to the final situation. One of the points of this paper is that this is not the case at all. Upon closer inspection, income changes in E1 give rise to two effects: a change in cross-section inequality from I(1, 3) to I(4, 3), and a permutation of the ordering of individual incomes between the first- and the second-period income distributions. In y_1 individual 1 is poorer than 2, while in y_2 individual 1 is richer.

At this point, it is useful to consider a third example, denoted by E3. Now the income structure is $\mathbf{Y}^* = \{(2, 4), (5, 2)\}$ and $y_a{}^* = (7, 6)$. Both the initial situation and the rate of income growth coincide with those of examples E1 and E2. Given the symmetry of I(.), we have that I(6, 7) = I(7, 6). Therefore, we have that $M(\mathbf{Y}^*) = M(\mathbf{Y})$. The (important) novelty in relation to E1, is that in E3 there is both a permutation between the two period distributions $y_1{}^*$ and $y_2{}^*$, and what we call a reranking between the first-period and the aggregate income distributions, $y_1{}^*$ and $y_a{}^*$, respectively.

Examples E1 and E3 suggest that our mobility index can be decomposed in two terms. One capturing the welfare change due to the change in inequality between the cross-section distributions y_1 and y_2 without any permutation, and a second one capturing the permutation effect with or without reranking between y_1 and y_a. In our opinion, this distinction follows closely the one found in the sociological literature between structural mobility and exchange mobility(14). Therefore, from a formal point of view what we wish to achieve is a decomposition of the mobility index $M(\cdot)$ into structural mobility $SM(\cdot)$

and exchange mobility EM(·). For that purpose, it is important to retain the following terminology. Given an income structure $\mathbf{Y} = (y_1, y_2) \in D^2$, we will always consider that y_1 is ordered according to the "less than or equal" relation. Whenever y_1 and y_2 are not equally ordered, we say that there has been some *permutation* between them; whenever y_1 and $y_a = y_1 + y_2$ are not equally ordered, we say that there has been some *reranking* between them. Of course, the reranking between y_1 and y_a implies the permutation between y_1 and y_2 (as in **E3**), but not the contrary (as in **E1**). Finally, given any income structure $\mathbf{Y} = (y_1, y_2) \in D^2$, define $y_c = y_1 + y_2'$, where y_2' is the second-period distribution y_2 ordered as the initial distribution y_1. Armed with these concepts, we suggest the following decomposition of our mobility index:

$$M(\mathbf{Y}) = SM(\mathbf{Y}) + EM(\mathbf{Y}),$$

where

$$SM(\mathbf{Y}) = \{W(y_c) - W(y_b)\} / W(y_b) = \{I(y_1) - I(y_c)\} / \{1 - I(y_1)\} \tag{5}$$

$$EM(\mathbf{Y}) = \{W(y_a) - W(y_c)\} / W(y_b) = \{I(y_c) - I(y_a)\} / \{1 - I(y_1)\}. \tag{6}$$

<u>Remark 1</u>. Since $I(y_2') = I(y_2)$ and

$$I(y_c) \in \{Min(I(y_1), I(y_2)), Max(I(y_1), I(y_2))\},$$

we have that

$$SM(\mathbf{Y}) \gtreqless 0 \Leftrightarrow I(y_1) \gtreqless I(y_2). \tag{7}$$

That is, the structural mobility captures the welfare change due to the change in cross-section inequality.

Consider the case in which there is no permutation between y_1 and y_2, so that $y_2' = y_2$, and $y_c = y_a$. In King (1983)' s model there is no mobility. In our case, all mobility is structural mobility which, by (7), in general it is different from zero.

In the presence of some permutation between y_1 and y_2, we can show that exchange mobility is always socially desirable (See Theorem 1. i)). Moreover, in a number of cases we can sign $M(\cdot)$. In the first place, if $I(y_1) \geq I(y_2)$, then by (7) structural mobility is non-negative. Hence overall mobility will be positive. An example of this situation is provided by E1, illustrated in Figure 1.

Figure 1 around here

When $I(y_1) < I(y_2)$, the sign of $M(\cdot)$ depends on the relative strength of $EM(\cdot)$ and $SM(\cdot)$. But if there is no reranking between y_1 and y_a we can show that $M(\cdot)$ is positive. Formally, we have:

Theorem 1. Let $Y = (y_1, y_2) \in D^2$ such that $y_2' \neq y_2$ and $y_c \neq y_a$, i.e. such that there is some permutation between y_1 and y_2.

i) $EM(Y) > 0$.

ii) If $I(y_1) \geq I(y_2)$ or there is no reranking between y_1 and y_a, then $M(Y) > 0$.

(See the Proof in the Appendix).

Consider the example E4, illustrated in Figure 2, where $Y^{\&} = \{(2, 4), (7, 0)\}$ and $y_a{}^{\&} = (9, 4)$. There is a reranking between $y_1{}^{\&}$ and $y_a{}^{\&}$ and, therefore, a permutation between $y_1{}^{\&}$ and $y_2{}^{\&}$ which causes $EM(Y^{\&}) > 0$. On the other hand, since $I(y_1{}^{\&}) < I(y_2{}^{\&})$ we have that $SM(Y^{\&}) < 0$. It turns out that the $SM(\cdot)$ is stronger than $EM(\cdot)$ so that $M(Y^{\&}) < 0$.

Figure 2 around here

In the presence of rerankings, we can show that there exists some reallocation of the second-period total income which gives rise to the same mobility but with no reranking at all. The elimination of rerankings does away with some or all permutations, causing the exchange mobility to decrease or to disappear altogether. Overall mobility remains constant because the new second-period income distribution has less inequality than the original one, a change that implies an increase in structural mobility which exactly offsets the reduction in exchange mobility. Formally, we have:

Theorem 2. Let $Y = (y_1, y_2) \in D^2$ so that there is some reranking between y_1 and y_a. Then, there exists some $\overset{*}{y_2} \in D$ with the following properties:

i) $\mu(\overset{*}{y_2}) = \mu(y_2)$; ii) $M(Y^*) = M(Y)$, where $Y^* = (y_1, \overset{*}{y_2})$;

iii) There is no reranking between y_1 and $\overset{*}{y_a} = y_1 + \overset{*}{y_2}$; iv) $I(\overset{*}{y_2}) < I(y_2)$.

(See the Proof in the Appendix).

If we are interested at all in the social welfare during the second period, then Theorem 2 ensures that, in the presence of rerankings, we can always increase the original second period welfare maintaining overall mobility constant.

The final question in this Section is the following: given an income structure $\mathbf{Y} = (y_1, y_2)$, what happens when we switch the roles of y_1 and y_2? The answer is that all depends on the relationship between $I(y_1)$ and $I(y_2)$: the smaller the inequality in the initial situation, the greater the income mobility. Formally, we have:

Remark 2. Let $\mathbf{Y} = (y_1, y_2)$ with $y_a = y_1 + y_2$. Assume, without loss of generality, that there is some permutation between y_1 and y_2, and let $y_c = y_1 + y_2'$. Define $\mathbf{Y}^* = (y_1^*, y_2^*)$ where $y_1^* = y_2$, ordered according to the "less than or equal" relation, and $y_2^* = y_1$ ordered so that $y_a^* = y_1^* + y_2^* = y_a'$. Therefore, $I(y_a^*) = I(y_a)$. Notice also that $y_c^* = y_c$. Then we have that

$$SM(\mathbf{Y}) = \{I(y_1) - I(y_c)\}/\{1 - I(y_1)\} \gtrless SM(\mathbf{Y}^*) = \{I(y_2) - I(y_c)\}/\{1 - I(y_2)\} \Leftrightarrow I(y_1)\} \gtrless I(y_2)$$

and

$$EM(\mathbf{Y}) = \{I(y_c) - I(y_a)\}/\{1 - I(y_1)\} \gtrless EM(^*) = \{I(y_c) - I(y_a)\}/\{1 - I(y_2)\} \Leftrightarrow I(y_1)\} \gtrless I(y_2).$$

Hence,

$$M(\mathbf{Y}) = SM(\mathbf{Y}) + EM(\mathbf{Y}) \gtrless EM(^*) = SM(\mathbf{Y}^*) + EM(\mathbf{Y}^*) \Leftrightarrow I(y_1)\} \gtrless I(y_2).$$

III. THE INCOME TAX MODEL

III. 1. The Homogeneous Case

Let us assume that we have a set of $i = 1,..., n$ homogeneous individuals that can only differ in their pre-tax income. Let us denote by $\mathbf{y} = (y^1,..., y^n)$ and $\mathbf{x} = (x^1,..., x^n)$ the pre-tax and the after-tax income distributions, respectively, and let $\mathbf{T} = (t^1,..., t^n)$ be the income tax vector. Then, $\mathbf{x} = \mathbf{y} - \mathbf{T}$. We say that a tax vector T is progressive, proportional or regressive in a relative sense according to whether $I(T) \geq I(y)$, respectively. We refer to $\mathbf{Y} = \{\mathbf{x}, \mathbf{T}\} \in D^2$ as an income-tax pair, where $\mathbf{x}$ is ordered by the "less tan or equal" relation.

In the terminology of the previous section, an income-tax pair is an income structure where the aggregate situation is seen to be $y_a = x + T = y$. Applying the definition given in equation (2) and taking into account assumption A. 5, The CDW measure of income mobility induced by the tax system is

$$M(Y) = \{W(y) - W(x_b)\} / W(x_b) = \{I(x) - I(y)\} / \{1 - I(x)\}. \qquad (8)$$

The numerator in (8) is the negative of the redistributive effect (RE, for short) usually defined as $RE(Y) = I(y) - I(x)$ in the income tax literature. Therefore, it seems convenient to change the definition in (8) to:

$$M(Y) = \{W(x) - W(y_b)\} / W(y_b) = \{I(y) - I(x)\} / \{1 - I(y)\}, \qquad (9)$$

where y_b is the hypothetical income distribution which would have resulted from a proportional income tax with the same tax revenue as T. According to equation (9), the income mobility induced by the tax system leads to a welfare improvement if and only if there is a positive RE, i.e., a reduction in the after tax income inequality.

Notice that, given $x = y - T$, any reranking between y and x implies the existence of permutations between x and T. However, if marginal tax rates are less than one, then it is impossible to have any rerankings between the pre-tax and the after-tax income distributions. Even without rerankings between y and x there can be permutations between x and T. But this would lead to permutations between y and T, implying that a poorer pre-tax individual pays a greater income tax than a richer one. In the present homogeneous word, we rule out such absolutely regressive tax systems. In this case we have that

$$I(y) \in \{\mathrm{Min}(I(x), I(T)), \mathrm{Max}(I(x), I(T))\},$$

so that

$$M(Y) \gtreqless 0 \Leftrightarrow RE(Y) \gtreqless 0 \Leftrightarrow I(T) \gtreqless I(x) \Leftrightarrow I(T) \gtreqless I(y). \qquad (10)$$

Equation (10) indicates that the sign of the RE -and hence the sign of the mobility index- depends on whether the tax vector is progressive, proportional or regressive, a well known result[15].

III.2. The Heterogeneous Case

In the real world, tax units may differ in income and/or non-income characteristics, like marriage status, number of dependents, income sources, housing tenure, or financial asset structure. Moreover, real life tax systems can be thought of as a pair consisting of a progressive tax tariff, and a complex set of exemptions, allowances, and tax credits. The effect of such a tax system on heterogeneous tax units may very well give rise to the thorny issues relating to horizontal inequality.

The principle of horizontal equality requires equal treatment of equals by the tax system. But the application of this principle is plagued with difficulties. In the first place, in a heterogeneous world there can be differences between the notions of equals used by the analyst and by the fiscal authority. Then, whatever the method used to measure horizontal inequities, we are likely to include in our estimates what we call "unintended horizontal inequality"(16). In the second place, independently of the notion of equals we care to use, we must confront the well known difficulty that, in the real world, we find very few identical tax units in the agreed upon space. One way to approach this difficulty is to couch the analysis in terms of "similars" rather than "exact equals"(17). An alternative approach consists of the identification of horizontal equality with the preservation of the pre-tax income distribution's ordering. In our notation, Plotnick (1982, 1985) and King (1983), for example, propose to measure horizontal inequality as the extent of rerankings between y and x. In our opinion, there can be unequal treatment of equals which does not give rise to rerankings. Nevertheless, it is clear that any reranking constitutes *prima facie* evidence of an horizontal inequity worth worrying about and measured for its own sake.

We know that rerankings between y and x imply permutations between x and T. But we have seen that there can be permutations between x and T without rerankings, in which case we have permutations between y and

T implying that a poorer pre-tax individual pays a greater income tax than a richer one. We will eventually distinguish between these two phenomena, but we must start by presenting the definitions of structural and exchange mobility. Given the income-tax pair $\mathbf{Y} = (\mathbf{x}, \mathbf{T}) \in D^2$, let $\mathbf{T}'$ be the vector $\mathbf{T}$ ordered as $\mathbf{x}$, and define $\mathbf{z} = \mathbf{x} + \mathbf{T}'$. Finally, let $\mathbf{z}_b$ be the hypothetical income distribution which, starting from the pre-tax vector $\mathbf{z}$, would have resulted from a proportional income tax with the same tax revenue as $\mathbf{T}$. Using these concepts, we propose the following decomposition

$$M(\mathbf{Y}) = SM(\mathbf{Y}) + EM(\mathbf{Y}),$$

where

$$SM(\mathbf{Y}) = \{W(\mathbf{x}) - W(\mathbf{z}_b)\}/W(\mathbf{y}_b) = \{I(\mathbf{z}) - I(\mathbf{x})\}/(1 - I(\mathbf{y})), \qquad (11)$$

and

$$EM(\mathbf{Y}) = \{W(\mathbf{z}_b) - W(\mathbf{y}_b)\}/W(\mathbf{y}_b) = \{I(\mathbf{y}) - I(\mathbf{z})\}\}/(1 - I(\mathbf{y})). \qquad (12)$$

Equation (11) measures the welfare change induced by the tax system if all permutations between x and T would have been eliminated in a hypothetical situation in which the pre-tax income distribution adjusts -becoming z- in order for the after-tax income distribution to remain equal to the original vector x. Equation (12) measures the welfare change induced by the two types of permutations between x and T we have discussed.

Remark. In general,

$$I(\mathbf{z}) \in \{\mathrm{Min}(I(\mathbf{x}), I(\mathbf{T})), \mathrm{Max}(I(\mathbf{x}), I(\mathbf{T}))\},$$

so that

$$SM(\mathbf{Y}) \gtreqless 0 \Leftrightarrow I(\mathbf{T}) \gtreqless I(\mathbf{x}). \qquad (13)$$

If there is no permutation between x and T, then $T' = T$, $z = y$, $EM(Y) = 0$, and $M(Y) = SM(Y)$, i. e., all mobility is structural mobility. In addition, since there is no permutation between y and T, not only is equation (13) satisfied but also equation (10) as in the homogeneous case.

If there is some permutation between x and T, because our definition of income mobility is given in equation (9) rather than (8), then by Theorem 1. i) we have that $EM(\cdot) < 0$, that is, in the income tax model permutations are always welfare decreasing. As in the income growth model, we can sign $M(\cdot)$ in a number of cases. In the first place, if $I(T) \leq I(x)$, then by (13) $SM(\cdot) \leq 0$ and, therefore, $M(\cdot) < 0$. In the second place, consider a case in which there is no reranking between x and y. For instance, consider the income-tax pair $Y^{\#} = \{x^{\#}, T^{\#}\} = \{(6, 12), (6, 2)\}$ with the pre-tax income distribution $y^{\#} = (12, 14)$. It is clear that $I(x^{\#}) > I(y^{\#})$, so that $M(Y^{\#}) < 0$. To understand this example, notice that $T^{\#\prime} = (2, 6)$ and $z^{\#} = x^{\#} + T^{\#\prime}$. Because $I(T^{\#}) > I(x^{\#})$, by (13), $SM(Y^{\#}) > 0$. However, the permutations involved in such an absolutely regressive tax vector cause a negative and large exchange mobility component which dominates the structural mobility effect. Of course, this example is but one instance of the application of Theorem 1. ii) to the income tax model.

Consider the following example, illustrated in Figure 3, of a reranking between x and y. The pre-tax income distribution is $y^{\&} = (9, 6)$. There is a progressive tariff which leads to the tariff vector $T_1 = (4, 2)$. There is also a vector of tax credits $C = (0, 1.5)$. Therefore the total or effective tax vector is $T^{\&}$

$= (4, 0.5)$ -a very progressive one- while the after-tax vector is $x^{\&} = (5, 5.5)$. Let $Y^{\&} = \{x^{\&}, T^{\&}\}$. Notice that $T^{\&\prime} = (0.5, 4)$ and $z^{\&} = x^{\&} + T^{\&\prime} = (5.5, 9.5)$. The adverse consequences of the permutation between $x^{\&}$ and $T^{\&}$ are reflected in the fact that $EM(Y^{\&}) < 0$. The positive welfare consequences of the progressivity of $T^{\&}$ are reflected in the fact that $SM(Y^{\&}) > 0$. This effect offsets the previous one, so that $M(Y^{\&}) > 0$, reflecting the positive RE we have learnt to expect from a progressive tax system, even in the (unwelcome) presence of rerankings between pre-tax and after-tax incomes.

Figure 3 about here

Of course, in other cases the exchange mobility can dominate the structural mobility yielding a negative overall mobility measurement. A key feature of our model is that we do not impose any value judgments on the deleterious effects of the rerankings between x and y induced by an income tax. But an application of Theorem 2 shows that, whenever there is some reranking between the pre-tax and the after-tax income distributions, there exists some new tax system T^* with the same tax revenue as T which gives rise to the same income mobility but generates no reranking at all.

This tax vector is defined by $T^* = y^{\&\prime} - x^{\&}$, where $y^{\&\prime}$ is the pre-tax vector ordered as $x^{\&}$. In the previous example, $T^* = (1, 3.5)$. If we now define $x^{\wedge} = y^{\&} - T^{\wedge}$, where $T^{\wedge}$ is the tax vector T^* ordered as $y^{\&}$, then we have that $x^{\wedge}$ is simply the original after-tax vector $x^{\&}$ ordered as $y^{\&}$. Thus, the income-tax pair $Y^{\wedge} = \{x^{\wedge}, T^{\wedge}\}$ with $y^{\wedge} = x^{\wedge} + T^{\wedge} = y^{\&}$, has the same income mobility as the original one, but without any reranking between $x^\wedge$ and $y^\&$. Notice that $T^\wedge$ is still progressive, but since $I(T^\wedge) < I(T^\&)$, we have eliminated what we may call the "excess progressivity" which was causing the reranking between $y^\&$ and $x^\&$. A consequence of this reduction in inequality is that the social welfare of the tax vector is increased: $W(T^\wedge) - W(T^\&) = \mu(T^\&)\,(I(T^\&) - I(T^\wedge)) > 0$. On the other hand, the progressivity reduction may have positive incentive effects on economic activity, an issue beyond the scope of this paper.

Finally, we are in a position to disentangle the effect of the two types of permutations between x and T in the income tax model. Consider an income-tax pair $Y = \{x, T\}$, with $y = x + T$, where there are some permutations between x and T. Let $M(Y) = SM(Y) + EM(Y)$. If there are no rerankings between x and y, then $EM(Y)$ measures how important are the consequences of the fact that the tax vector T is absolutely regressive. If there are rerankings between x and y, then $EM(Y)$ may capture the impact of both types of permutations. Let $Y^\wedge = \{x^\wedge, T^\wedge\}$, with $y = x^\wedge + T^\wedge$ be the income-tax pair in which all rerankings have been eliminated after applying Theorem 2. If there are still some permutations between $x^\wedge$ and $T^\wedge$, then $M(Y^\wedge) = SM(Y^\wedge) + EM(Y^\wedge)$, where $EM(Y^\wedge)$ measures the effect of permutations due solely to the fact that $T^\wedge$ is absolutely regressive. In this case, the difference between $EM(Y)$ and $EM(Y^\wedge)$ allows us to estimate the exchange mobility due exclusively to the reranking between x and y caused by the excessively progressive tax vector T. Of course, if there are no permutations between $x^\wedge$ and $T^\wedge$, then $EM(Y^\wedge) = 0$, $M(Y^\wedge) = SM(Y^\wedge)$ and $EM(Y)$ provides a direct measure of the extent of the rerankings.

IV. OTHER APPLICATIONS

In this section we will briefly describe other applications of these two-period models.

IV. 1. The Impact of Different Assumptions About Equivalence Scales

As we have pointed out in Section III, in the real world we have information about a set of heterogeneous individuals -tax units or households- with different characteristics and different needs. In income distribution theory, one usually takes into account different needs due to different demographic characteristics. For simplicity, in what follows we only consider the household size. Units of the same size are assumed to have the same needs and, therefore, their incomes are directly comparable. However, social evaluation within individual subgroups need not yield unanimous results. Moreover, it is always convenient to extract conclusions for the population as a whole. Therefore, we need a procedure to establish welfare comparisons for households of different size. This is, of course, the role played by equivalence scales.

We assume that larger units have greater needs, but also greater opportunities to achieve economies of scale in consumption. Assume that there are $k = 1,...,K$ unit sizes. Following Buhman *et al.* (1988) and Coulter *et al.* (1992a, 1992b), for each household i of size k define adjusted income in the relative case by

$$y^i(\Theta) = y^i / k^{\Theta},\ \Theta \in [0,1]. \tag{14}$$

Taking a single adult as the reference type, the expression k^{Θ} can be interpreted as the number of equivalent adults in a household of size k. Thus, the greater is Θ, the greater the number of equivalent adults in each household or, in other words, the smaller the economies of scale. When $\Theta = 0$ and economies of scale are assumed to be infinite, adjusted income coincides with unadjusted household income; while if $\Theta = 1$ and economies of scale are completely ruled out, then adjusted income equals *per capita* household income.

According to the empirical literature, the inequality of adjusted income follows a U pattern as a function of Θ(18). However, these are not the only changes which take place: the relative positions of units of different size are drastically altered as Θ varies from 0 to 1. The reason is found in the positive association we observe between income and unit size. Thus, when economies of scale are assumed to be infinite and $\Theta = 0$, single person units tend to be poorer relative to large ones. The opposite is the case as economies of scale lose importance when Θ rises toward 1. It is well known that this reordering may influence decisively the study of poverty as well as international comparisons of inequality in the presence of large differences in demographic characteristics(19). The question we want to address here is: how can we measure the welfare effect caused by such reorderings?

Let $\Theta_2 > \Theta_1$. Taking into account (14), for each i with $k^i \geq 2$ adjusted income for Θ_2 is smaller than for Θ_1. Let us denote the difference by e^i. If we denote the corresponding vectors by $y(\Theta_2)$, $y(\Theta_1)$ and e, then we have that $y(\Theta_2) = y(\Theta_1) - e$. Therefore we can apply the analysis developed for the income tax model to the pair $Y = \{y(\Theta_1), e\}$.

IV. 2. Tax-benefit Models

We suggest reinterpreting the income tax model in a situation where we have microeconomic information on both tax and public benefits for a set of individuals. Let y be the vector of benefits, T the vector of taxes, and x the vector of net benefits where $x = y - T$. We may call $Y = \{x, T\}$ a tax-benefit system. As in the income tax model, we are interested in rerankings between y and x which cause permutations between x and T. But there can be other permutations between x and T due to the fact that some individuals who receive low net benefits are paying larger taxes than other individuals receiving high net benefits. The index $M(\cdot)$ defined in equation (9) measures the income mobility due to the tax benefit system as a whole. The decomposition in equations (11) and (12) is useful to measure the importance of the rerankings between gross and net benefits induced by the tax system.

To deal with the problem that x may involve individuals with negative net benefits, we may use a SEF which can be expressed as the difference between the mean and an index of absolute inequality (See note 13).

Alternatively, we may partition the sample into those individuals with positive and negative net benefits. To analyze the second group, we may consider a model with the vector of net taxes **t** = **T** - **y** as the reference vector, that is, an income structure **Y** = {**t**, **y**}.

Typically, we are also interested in the impact of net benefits on the distribution of income before the intervention of the public sector. We can study this problem with the help of the income growth model developed in Section II. Let y_1 be the income distribution before public benefits and taxes, and let y_2 be the vector of net benefits (called x in the previous application). Then $Y = \{y_1, y_2\}$ with the aggregate or the final situation given by $y_a = y_1 + y_2$. Our concepts permit to study both the impact of differences in income inequality between y_1 and y_2, as well as the effect of permutations between these two distributions -with or without rerankings between the "private" income distribution y_1 and the net benefit distribution y_2.

IV. 3. Different Income Sources

In income distribution theory, we are often interested in evaluating the distributional implications of adding up two different income sources. For example, let us denote by y_1 the earnings distribution of household heads. We want to know the consequences of adding up the spouses earnings included in vector y_2. In our terminology, $Y = \{y_1, y_2\}$ constitutes an income structure. The mobility index provides a measure of the welfare effect of adding up the earnings of household heads and their spouses. In terms of our model, there are two forces influencing the sign and magnitude of M(Y): i) the difference in the earnings inequality of the two groups, and ii) the impact of permutations between y_1 and y_2, with or without reranking between y_1 and $y_a = y_1 + y_2$.

IV. 4. Dynasties

In income mobility theory, what we are often interested in is the extent to which parents determine the positions occupied by their sons and daughters. Assume we have a procedure to express a person's life cycle income stream by means of a scalar. Let y_1 be the parents life cycle income distribution, and let y_2 be the descendants life cycle income distribution. Again, $Y = \{y_1, y_2\}$ constitutes an income structure to which we can apply our concepts. In this model, $y_a = y_1 + y_2$ can be interpreted as the dynastic income distribution. Quite naturally, income mobility arises from a comparison between the welfare of the dynasties in the actual income structure, $W(y_a)$, and the welfare of the dynasties in a hypothetical benchmark structure Y_b: the income structure which would have resulted if descendants life cycle incomes have the same inequality as their parents life cycle incomes.

V. CONCLUSIONS

The literature on measures of relative income mobility studies two types of income changes which can be observed with longitudinal data: changes in cross-section income inequality, and changes in relative incomes.

Within the limits of a two period model, we have suggested a way to decompose CDW ethical index of relative mobility into an index of structural mobility and an index of exchange mobility which capture the welfare effect of these two types of income changes. In so doing, we have shown the relevance of distinguishing between two types of reorderings: permutations between the first- and second-period income distributions, and rerankings between the initial and the aggregate or final situation.

We have used these indices to study the changes induced by an income tax in a heterogeneous world in which tax units may differ in income and/or non-income characteristics. We have shown that our mobility index has the same sign that the redistributive effect in the income tax literature, namely, the difference between pre-tax and after-tax income inequality. Our exchange mobility index can be used to measure the extent of rerankings between the pre-tax and after-tax income distributions, which is viewed by some authors as a measure of horizontal inequality.

In the income growth model, exchange mobility is always welfare enhancing, while the opposite is the case in the income tax model. Also, in the presence of rerankings we have shown that there exists a second-period income distribution which generates the same effects as the original one but involves no reranking at all. In the income tax context, this implies that it is always possible to eliminate the horizontal inequities without detracting from the redistributive effect and the tax revenue of the original tax system.

From a conceptual point of view, we should emphasize that all of the above has been accomplished in the framework chosen by CDW which, contrary to the seminal contribution by King (1983), does not involve any new value judgments on either permutations or rerankings. In particular, we do not put positive value on rerankings in an income growth context, nor negative value when the rerankings are induced by an excessively progressive income tax.

We believe that in problems where there are individual rank reversals, our approach is immediately applicable. We have shown that there are a number of interesting applications even in the simple two dimensional models developed in this paper. However, the greatest limitation of this approach is possibly the restriction to a two period world.

The extension to a truly multiperiod context must start with a model of how to evaluate, from an ethical point of view, a multiperiod income stream at the individual level. On the other hand, if the present two period model were to be naively extended to three or more periods, we know that the results depend on the decision about the reference period. Therefore, one would have to come up with an appropiate suggestion for the notion of an immobile income structure in a multiperiod context.

Once these difficulties are solved, a multiperiod model can be applied to other problems which involve rank reversals. A possible dynamic application would be the measurement of convergence between countries or regions. In a static context, we may extend the analysis suggested in Section IV. 3 and IV. 4. to any number of income sources and dinasties, respectively.

APPENDIX

Theorem 1. Let $Y = (y_1, y_2) \in D^2$ so that $y_2' \neq y_2$ and $y_c \neq y_a$, i.e. so that there is some permutation between y_1 and y_2. i) $EM(Y) > 0$. ii) If $I(y_1) \geq I(y_2)$ or there is no reranking between y_1 and y_a, then $M(Y) > 0$.

Proof of i):

That there is some permutation between y_1 and y_2, means that there exists at least a pair of individuals $j < k$, such that $y_1^j \leq y_1^k$ but $y_2^j > y_2^k$. Let y_2' be the vector y_2 but ordered as y_1, so that $y_2^{j'} = y_2^k$ and $y_2^{k'} = y_2^j$. Since $y_c = y_1 + y_2'$, we have that

$$y_c^j = y_1^j + y_2^k \quad \text{and} \quad y_c^k = y_1^k + y_2^j .$$

Recall that $y_a^j = y_1^j + y_2^j$ and $y_a^k = y_1^k + y_2^k$. Therefore, $y_a^j - y_c^k = y_1^j - y_1^k < 0$ and $y_a^k - y_c^j = y_1^k - y_1^j > 0$, which implies that

$$y_a^j < y_c^k \quad \text{and} \quad y_a^k > y_c^j . \tag{1}$$

At the same time, $y_a^j - y_c^j = y_2^j - y_2^k > 0$ and $y_a^k - y_c^k = y_2^k - y_2^j < 0$, which implies that

$$y_a^j > y_c^j \quad \text{and} \quad y_a^k < y_c^k .$$

Suppose that there is a reranking involving individuals j and k. Then, given that $y_1^j < y_1^k$, we must have that $y_a^j \geq y_a^k$. By (1), we have:

$$y_c^j < y_a^k \leq y_a^j < y_c^k .$$

Suppose now that there is no reranking involving individuals j and k, so that $y_a^j \leq y_a^k$. By (2) we have

$$y_c^j < y_a^j \leq y_a^k < y_c^k .$$

In both cases, we have $I(y_a) < I(y_c)$, so that $EM(Y) = \{I(y_c) - I(y_a)\}/\{1 - I(y_1)\} > 0$.

Proof of ii):

That $I(y_1) \geq I(y_2)$ implies that $M(Y) > 0$ was shown in the text. Assume now that there is some permutation between y_1 and y_2 but no reranking between y_1 and y_a. This means that there exists at least a pair of individuals $j < k$, such that $y_1^j \leq y_1^k$, $y_2^j > y_2^k$ but $y_a^j < y_a^k$. Since $y_a^j = y_1^j + y_2^j$ and $y_a^k = y_1^k + y_2^k$, we have that

$$y_a^k - y_a^j = (y_1^k - y_1^j) + (y_2^k - y_2^j).$$

Therefore

$$y_a^k - y_a^j < (y_1^k - y_1^j),$$

and hence $I(y_a) < I(y_1)$. We conclude that

$$M() = (I(y_1) - I(y_a))/(1 - I(y_1)) > 0.$$

Q.E.D.

Theorem 2. Let $Y = (y_1, y_2) \in D^2$ so that there is some reranking between y_1 and y_a. Then, there exists some $y_2^* \in D$ with the following properties:

i) $\mu(y_2^*) = \mu(y_2)$; ii) $M(Y^*) = M(Y)$, where $Y^* = (y_1, y_2^*)$;

iii) There is no reranking between y_1 and $y_a^* = y_1 + y_2^*$; iv) $I(y_2^*) < I(y_2)$.

Proof:

Let y_a' be the vector y_a ordered as y_1. Define $y_2^* = y_a' - y_1$. If $y_a^{i'} = y_a^i$, then $y_2^{i*} = y_a^i - y_1^i = y_2^i > 0$. If $y_a^{i'} \neq y_a^i$, then $y_a^{i'} = y_1^l + y_2^l$ for some $l > i$. Since $y_1^i \leq y_1^l$, $y_2^{i*} = y_1^l + y_2^l - y_1^i > 0$. Thus, $y_2^* \in D$, so that $Y^* = (y_1, y_2^*)$ is an income structure with $y_a^* = y_1 + y_2^* = y_a'$.

Since $\mu(y_a^*) = \mu(y_a') = \mu(y_a)$, we have $\mu(y_2^*) = \mu(y_2)$, which is condition (i). Since $I(y_a^*) = I(y_a') = I(y_a)$, we have

$$M(Y^*) = \{I(y_1) - I(y_a^*)\}/\{1 - I(y_1)\} = \{I(y_1) - I(y_a)\}/\{1 - I(y_1)\} = M(Y),$$

which is condition (ii). Since $y_a^* = y_a'$ and y_a' is ordered as y_1, there is no reranking between y_1 and y_a^*, which is condition (iii). That there is some reranking between y_1 and y_a, means that there exists at least a pair of individuals $j < k$, such that $y_1^j < y_1^k$ but $y_a^j = y_1^j + y_2^j > y_a^k = y_1^k + y_2^k$. Therefore,

$$y_2^j - y_2^k > y_1^k - y_1^j. \tag{3}$$

Since by (iii) there is no reranking between y_1 and y_a^*, we have that $y_1^j + y_2^{j*} = y_a^{j'} < y_1^k + y_2^{k*} = y_a^{k'}$. Therefore,

$$y_1^k - y_1^j > y_2^{j*} - y_2^{k*}. \tag{4}$$

By (3) and (4):

$$y_2^j - y_2^k > y_2^{j*} - y_2^{k*}.$$

Thus, $I(y_2) > I(y_2^*)$, which is condition (iv).

Q.E.D.

NOTES

(1) For an illuminating account of the main features of industrial society, see Gellner (1983, 1994).

(2) Among the descriptive measures, see Shorrocks (1978a), Geweke *et al* (1986), and Conlisk (1990); among the normative ones, see Atkinson (1983), Markandya (1982, 1984), Conlisk (1989) and Dardadoni (1993). In most cases, transition matrices are assumed to follow a Markov chain, a property often rejected in empirical analysis (see Fields and Ok (1996) for references to the empirical literature).

(3) See Hart (1976), Shorrocks (1993), and Conlisk (1974).

(4) For descriptive measures, see the relative indices suggested by Shorrocks (1978b) and Cowell (1985), and the absolute indices due to Berrebi and Silber (1983) and Fields and Ok (1996).

(5) For other versions of S(.) see, for instance, Maasoumi and Zandvakili (1989, 1990), based on Maasoumi (1986), as well as the criticism of them by Dardadoni (1990). For another approach to the construction of lifetime income, see Cowell (1979).

(6) In the absolute case, the benchmark structure Y_b would be chosen to be absolutely immobile, i.e. income differences would be preserved through time.

(7) In the absolute case, we would have $\overset{*}{M}_A(Y) = \Phi\{W(y_a) - W(y_b)\}$,

where $\Phi: R^1_{++} \rightarrow R^1$ is a continous increasing function with $\Phi(0) = 0$.

(8) In their discussion about the proper unit of egalitarian concern, McKerley (1989) and Tempkin (1992) call assumption 2 the *complete lives view*. By means of similar examples, they confront this approach with two other alternatives, including the *simultaneous lives view* which takes only into account the sequence of cross-section income distributions. From this perspective, **Y** would be preferable to **Y#**.

(9) Shorrocks (1978a) justifies **A. 4.** as a direct application in the intertemporal context of the population replication axiom, usually assumed in income distribution theory in order to compare the income inequality of populations of different size.

(10) A SEF function $W(\cdot)$ is weakly-homothetic if and only if for all income distributions $x, y \in D$ with the same mean, $W(x) \geq W(y) \Leftrightarrow W(\alpha x) \geq W(\alpha y)$ for all $\alpha > 0$. In the absolute case, the SEF $W(\cdot)$ is expressed in terms of the mean and a translatable index of absolute inequality. When $W(\cdot)$ is regular, this is the case if and only if $W(\cdot)$ is increasing along the rays parallel to the line of equality and weakly-translatable. A SEF is weakly-translatable if and only if for all income distributions $x, y \in D$ with the same mean, $W(x) \geq W(y) \Leftrightarrow W(x + \lambda e) \geq W(y + \lambda e)$, where **e** is a vector of ones and λ is such that $(x + \lambda e), (y + \lambda e) \in D$. (See Dutta and Esteban (1992)).

(11) See, for instance, Blackorby and Donaldson (1978).

(12) See Herrero and Villar (1989) and Ruiz-Castillo (1995a).

(13) In the absolute case, we would choose $\Phi(s) = s$ so that $M_A(Y) = W(y_a) - W(y_b)$. If we assume that W(.) is regular, increasing along rays parallel

to the ray of equality, and translatable, then $W(y) = \mu(y) - I^{KBD}(y)$, where $I^{KBD}(.)$ is the absolute inequality index obtained according to the Kolm-Blackorby-Donaldson procedure (See Kolm (1976a, 1976b) and Blackorby and Donaldson(1980)). Because of its decomposability properties, for operational purposes we would choose the Kolm-Pollak family of SEFs, $W_d^{KP}(.)$, where d is a parameter reflecting different degrees of aversion to absolute inequality. In this case, $W_d^{KP}(y) = \mu(y) - I_d^{KP}(y)$, where $I_d^{KP}(.)$ is the Kolm-Pollak index of absolute inequality consistent with $W_d^{KP}(.)$ (See Blackorby and Donaldson (1980)). In either case, the absolute mobility index would be $M_A(Y) = I_A(y_1) - I_A(y_b)$, with $I_A(.)$ equal to $I^{KBD}(.)$ or $I_d^{KP}(.)$.

(14) See the discussion about this notions in Markandya (1984) and Shorrocks (1993), and the references to the sociological literature quoted there.

(15) In the relative case, see the seminal paper by Jacobsson (1976) or Pfingsten (1988) and the references quoted there. In the absolute case, see Moyes (1988).

(16) For our contribution to this debate, see Ruiz-Castillo and Vargas (1997).

(17) See, for example, Berliant and Strauss (1985), Aronson et al. (1994) and Ruiz-Castillo and Vargas (1997).

(18) This is indeed the pattern reported by Coulter et al. (1992a, b) for the UK, by Rodrigues (1993) for Portugal, and by Ruiz-Castillo (1995b, 1998) for Spain.

(19) For the impact on poverty, see Lanjouw and Ravallion (1995) and Del Río and Ruiz-Castillo (1997). For the impact on international comparisons of inequality, see Burkhauser et al. (1996) and Garner et al. (1997).

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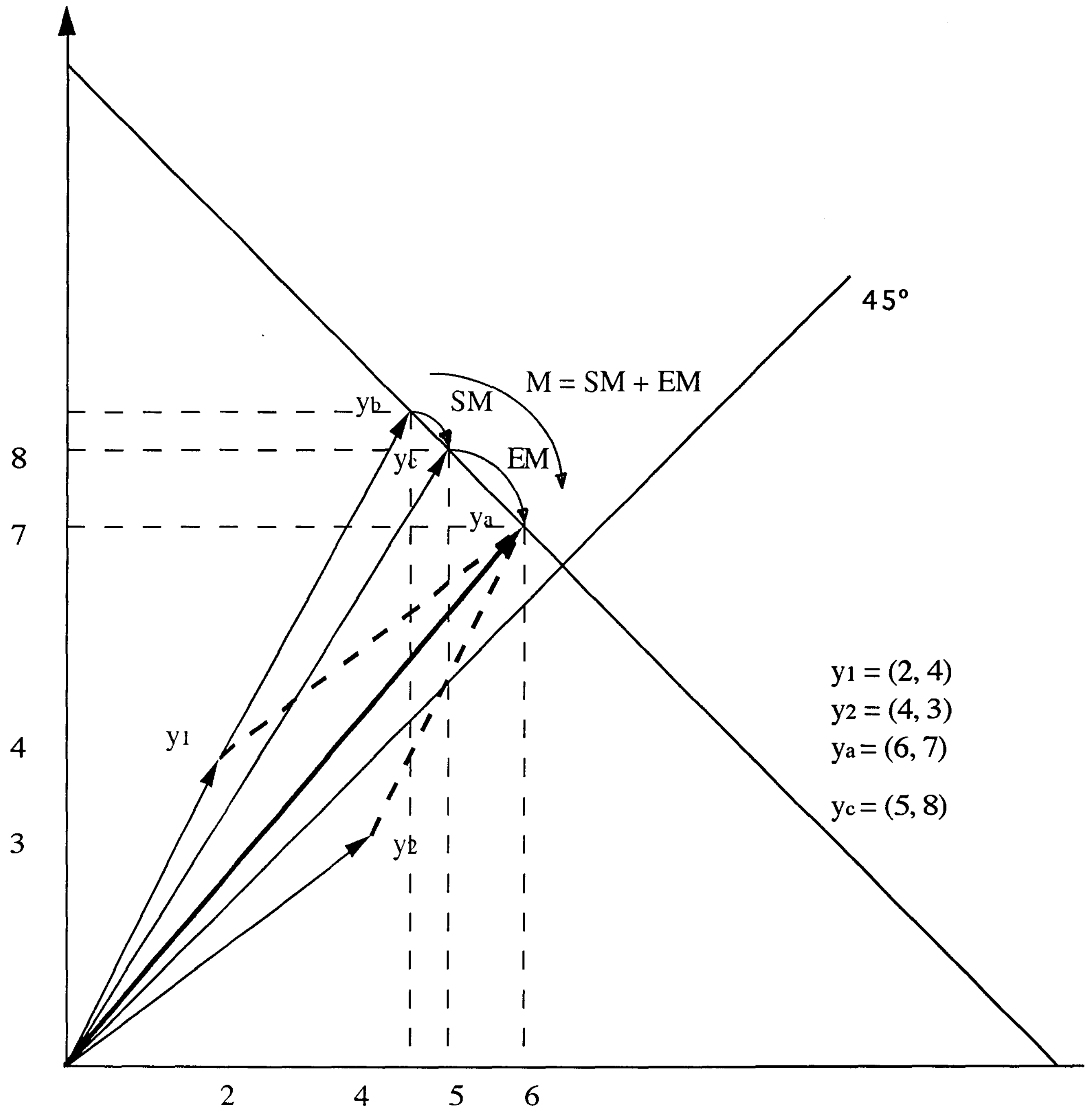
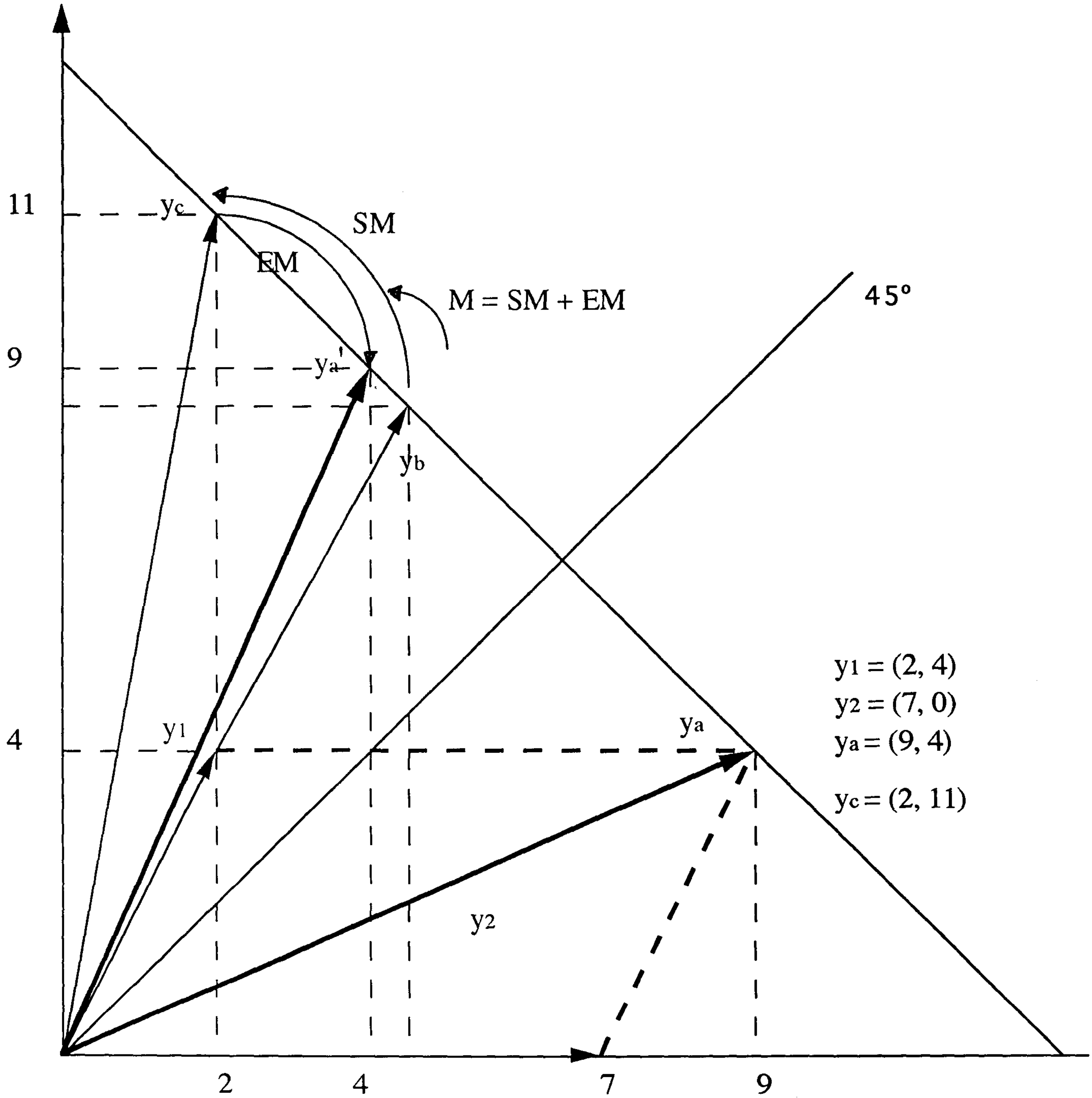


Structural mobility = $I(y_b) - I(y_c) > 0$

Mobility = $I(y_b) - I(y_a) > 0$

Exchange mobility = $I(y_c) - I(y_a) > 0$

FIGURE 1



Structural mobility = $I(y_b) - I(y_c) < 0$

Mobility = $I(y_b) - I(y_a) < 0$

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FIGURE 2

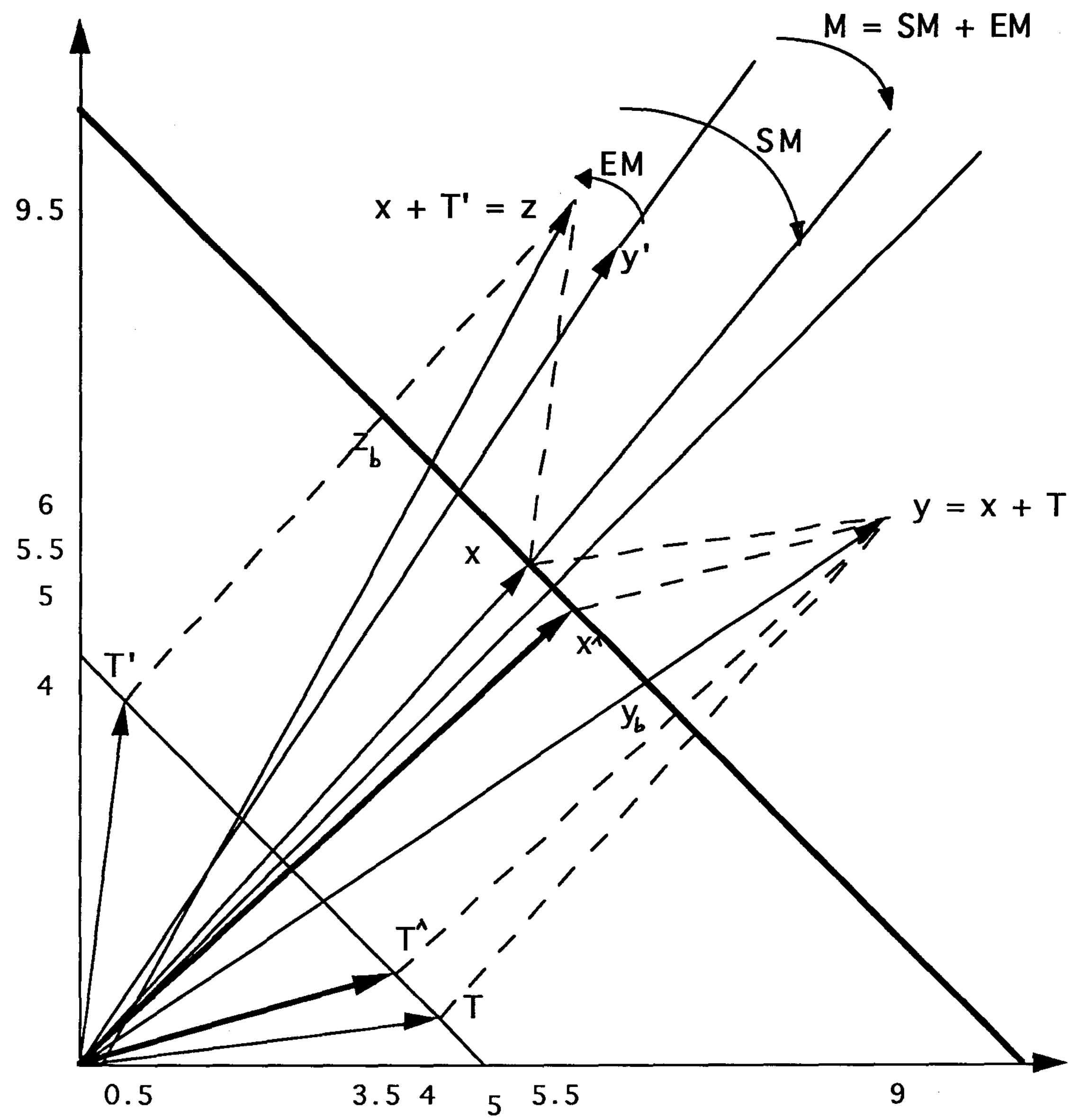

FIGURE 3

y = pre-tax income distribution = (9, 6) T = tax vector = (4, 0.5) x = y - T = after-tax income distribution = (5, 5.5) T' = tax vector ordered as x = (0.5, 4) z = x + T' = (5.5, 9.5) y' = pre-tax vector ordered as x

T^ = tax vector which generates the same tax revenue and income mobility but no rerankings = (3.5, 1) x^ = after tax vector ordered as y